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## ANALYSIS OF SOME PRELIMINARY LOW-LEVEL CONSTANT LEVEL BALLOON (TETROON) FLIGHTS<sup>1</sup>

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### ABSTRACT

An analysis is presented of low-level trajectory data obtained by means of constant level balloon flights from Cape Hatteras, N.C., during September and October 1959. An approximately constant floating level was obtained by flying the nearly constant volume Mylar balloons (tetroons) with an internal superpressure of about 100 mb. The metalized tetroons were positioned at 1-min. intervals by means of a manually operated SP-1M radar. From knowledge of these positions, overlapping 5-min. average velocities were determined for flight durations of up to 5 hr. On some of the flights the radar return was enhanced by the addition of a radar reflective mesh to the tetroon. With the addition of this mesh, flights at altitudes of less than 5,000 ft. were tracked as far as 92 n. mi. from the radar, or approximately to the radar horizon.

Spectral analysis of the velocity data obtained from the four best flights shows some evidence for a (Lagrangian) wind speed periodicity of 26-min. period, a vertical motion periodicity of 13-min. period, and a cross-stream velocity periodicity of 17-min. period. Cross spectrum analysis shows that, with the exception of oscillations of 45-min. period, the wind speed is at a maximum ahead of the trough in the trajectory. Thus, if the large-scale air flow is nearly geostrophic, there is evidence that kinetic energy and momentum are transported down the pressure gradient by these small-scale oscillations. The maximum upward motion of the tetroon tends to take place near the trajectory trough line for oscillations of a period exceeding 18 min. and near the trajectory crest for oscillations of smaller period. Therefore, looking downstream, the longer-period tetroon oscillations tend to be counterclockwise in a plane normal to the mean trajectory while the shorter-period oscillations tend to be clockwise. However, until more information is obtained on the small-scale temperature field and the extent to which the tetroons follow the vertical air motion, any statement regarding "direct" and "indirect" air circulations is tentative.

The ratio of one minus the cross-stream and one minus the along-stream autocorrelation coefficients for these flights is approximately 0.6, suggesting a certain similarity between Eulerian space and Lagrangian autocorrelation coefficients. The tetroon data also indicate that initially one minus the cross-stream autocorrelation coefficient is proportional to the time, as would be anticipated from Lagrangian turbulence theory. These data tend to confirm that, for the scale of motion under consideration, the Lagrangian-Eulerian scale factor  $\beta$  of Hay and Pasquill has a value near 4.

### 1. INTRODUCTION

Constant level balloons offer a means for approximating the trajectory of an air parcel in space. The transosonde program of the United States Navy has demonstrated

the usefulness of such balloons for the delineation of the large-scale air flow at jet stream levels over a large segment of the Northern Hemisphere [1, 2]. The purpose of this article is to show how constant level balloons may be utilized for the delineation of the small-scale air flow at levels near the surface and at distances up to 100 n. mi. from a radar site. The Lagrangian fluctuations derived

<sup>1</sup> Work performed in connection with Weather Bureau research for the U.S. Atomic Energy Commission and the U.S. Public Health Service.

from such low-level flights are of great interest in turbulence and diffusion studies.

## 2. THE SUPERPRESSURED BALLOON CONCEPT

Previous attempts to obtain the Lagrangian characteristics of the air flow at low levels have, in most cases, been based on the use of "neutral" or "no-lift" elastic balloons. While information of interest has been obtained from such flights [7], they have not proved particularly suitable for the determination of the Lagrangian characteristics of the flow over relatively long distances. Since constant volume, superpressured balloons have the capability of floating along a constant density surface almost indefinitely (barring large vertical air motions), it is appropriate to consider their use for this problem.

A balloon ascends or descends in the atmosphere according to whether its bouyancy force,  $V_b(\rho_a - \rho_h)$ , (where  $V_b$  is balloon volume,  $\rho_a$  is air density, and  $\rho_h$  is the density of a gas lighter than air, in this instance, helium) exceeds or is less than the weight,  $W_b$ , of the balloon system. However, if the balloon is of constant volume then, for a given mass of helium introduced into the balloon at the earth's surface, there is a certain air density where the bouyancy force and the weight of the balloon system balance. The 3-dimensional surface along which the atmosphere possesses this density is the surface along which the constant volume balloon will float. The balloon will continue to float along this surface as long as it remains at full volume. The balloon may go slack owing to seepage of helium through the skin of the balloon, through nighttime cooling of the balloon and inclosed helium due to radiation fluxes, or through cooling of the balloon due to any other causes. For these reasons it is desirable that initially, at flight level, the helium within the balloon exert a greater pressure than that of the ambient air. This pressure excess is known as superpressure. Analytically, the helium pressure ( $p_h$ ) required to prevent a slack balloon at the cold temperature  $T_c$  is given by

$$p_h = (T_w/T_c)p_a \quad (1)$$

where  $p_a$  is the ambient air pressure and  $T_w$  is the warm temperature. For example, if one assumes that a constant volume balloon flying at 900 mb. possesses a temperature of 300° A. during the day and a temperature of 280° A. during the night, then a daytime superpressure of 63 mb. would suffice to keep the balloon fully inflated at night. While statistics on day-night balloon temperatures at low flight levels are not yet available, one would expect that a daytime superpressure of 100 mb. would be sufficient to carry the balloon through the nighttime hours with no deviation from the density surface.

## 3. EQUIPMENT AND FLIGHT TECHNIQUES

A relatively new polyester film called Mylar<sup>2</sup> is highly

<sup>2</sup> Mylar—a trade name for a polyester film manufactured by the E. I. du Pont Company of Wilmington, Del.

suitable for use in the construction of constant volume balloons since it possesses very low permeability and high tensile strength (13,000 lb. in.<sup>-2</sup>). It can be shown that a spherical Mylar balloon of 1-foot radius and a skin thickness of only 2 mils (2/1000 in.) is capable of supporting a superpressure of 150 mb., thus undoubtedly satisfying the day-night requirements mentioned above. Furthermore, while it was originally feared that the seals or gores on the balloon would be unable to withstand such a superpressure, it was found that the G. T. Schjeldahl Company of Northfield, Minn. had perfected a method of heat sealing which yielded a seal strength comparable to that of the Mylar itself. For purposes of economy and reliability, however, it was desirable that these seals be straight lines. Therefore, tetrahedron-shaped balloons rather than spherical balloons were constructed for our experiments. Combination of the word "tetrahedron" and the word "balloon" results in the word "tetroon" as a designator for these particular balloons.

Tetroons with a side length of 36, 42, and 60 in. (nominal volumes, respectively, of 0.20, 0.32, 0.94 m.<sup>3</sup>) have been used in our experiments. It is desirable that the tetroon volume not exceed about 3 m.<sup>3</sup> partially in order to comply with regulations of the Federal Aviation Agency concerning the flying of balloons within air space. With no weight attached, the ceiling of the 42-in. tetroon is about 10,000 ft. while the ceiling of the 60-in. tetroon is about 20,000 ft. Figure 1 shows a picture of a 42-in. tetroon with inflation rig and manometer (for measuring superpressure) attached. Note that the tetroon has been coated with a molecular film of aluminum in order to provide a radar target.

Some of the earlier tetroon flights suggested that the tetroon volume was not being conserved under conditions of considerable superpressure, since the tetroons were flying at a higher elevation than would be anticipated from the balance of bouyancy force and weight on the assumption of constant balloon volume. If the tetroon volume is not absolutely constant as it ascends through the atmosphere it can be seen, by equating the weight of the balloon system and the bouyancy force, that the weight,  $X$ , which must be attached to the tetroon to enable it to fly at a given density surface is

$$X = V_e(\rho_a - \rho_h) - W_b = V_e\rho_a - V_s\rho_{hs} - W_b \quad (2)$$

where  $V_s$  is the tetroon volume at the earth's surface,  $V_e = V_s + \Delta V$  is the tetroon volume at flight level,  $\rho_a$  is the air density at flight level,  $\rho_h$  is the helium density at flight level,  $\rho_{hs}$  is the helium density at the earth's surface, and  $W_b$  is the weight of the balloon system.

Differentiating equation (2) with respect to pressure it is found that

$$\frac{\delta X}{\delta p} = V_e \frac{\delta \rho_a}{\delta p} + \rho_a \frac{\delta V_e}{\delta p} \quad (3)$$

The right hand term in this equation is negative since

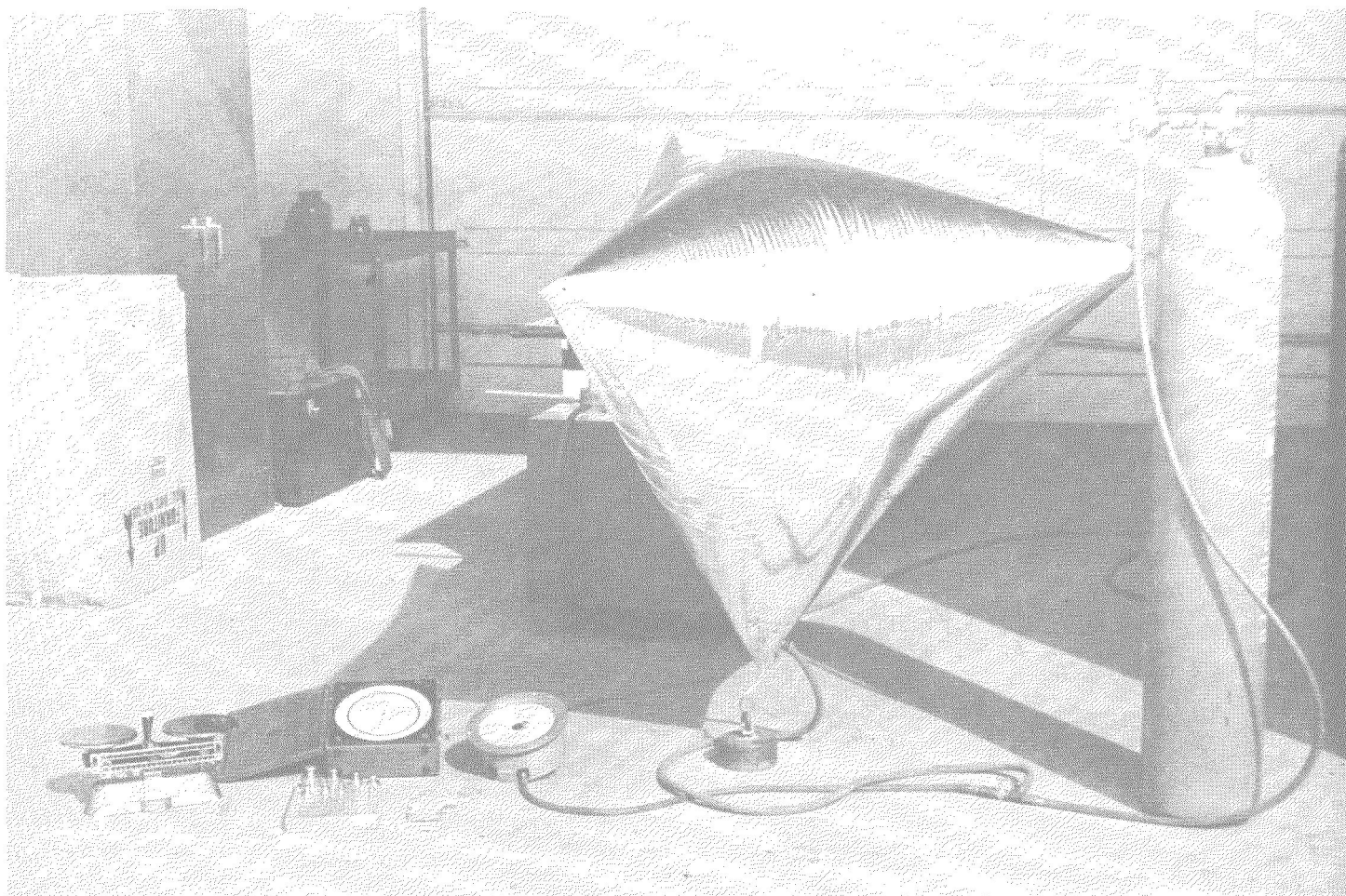


FIGURE 1.—Photograph of 42-in. tetron with inflation rig and manometer attached.

tetron volume increases with decrease in ambient pressure (increase in superpressure). If this negative term becomes larger in magnitude than the middle term in equation (3), the equation states that more weight must be attached to the tetron to make it float at a low pressure (high level) than at a high pressure (low level). Under these conditions the tetron is not in stable equilibrium at any level but continuously ascends, thus in no sense performing the function of a constant level balloon. This takes place when the volume increase of the tetron associated with increase in superpressure more than offsets the decrease in bouyancy force brought about by the decrease, with decrease in ambient air pressure, of the density difference between helium and air. It is apparent that the change of tetron volume with increase in superpressure must be determined precisely, not only to permit the flying of the tetroons at the required altitude, but also to ensure that a constant level balloon flight actually results.

The change in tetron volume as a function of superpressure was determined by careful weigh-off at the ground. The amount of superpressure within the tetron was measured by a sensitive manometer while the tetron

volume,  $V_b$ , was determined from the equation

$$V_b = \frac{W_b + L}{\rho_{as} - \rho_{hs}} \quad (4)$$

where  $L$  is the free lift of the tetron (measured by sensitive scales),  $\rho_{as}$  is surface air density, and the other parameters are as defined previously. Each of the tetroons tested was taken in steps up to a superpressure of 150 mb. with the tetron volume being determined at each step. The tetroons were then deflated (a vacuum cleaner is suitable, and necessary, for this) to near zero superpressure in order to note whether the volume change persisted. It was found that the tetroons are semi-elastic with about one-half the volume change remaining after deflation to a small superpressure. Figure 2 shows, by small circles, the 42-in. tetron volume change as a function of superpressure when the tetroons are first brought up to a superpressure of 150 mb. The analytic expression approximately fitting these circles is

$$\Delta V = 0.0038(e^{0.015\Delta p} - 1) \quad (5)$$

where  $\Delta V$  is the volume change in cubic meters and  $\Delta p$

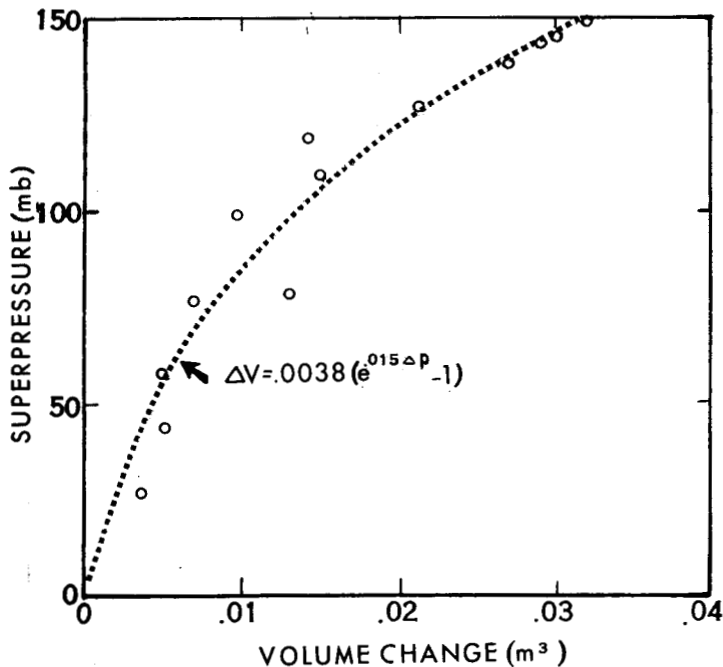


FIGURE 2.—Volume changes of 42-in. tetrons as a function of tetron superpressure (circles) and the fit of an analytic expression to these values (dotted line).

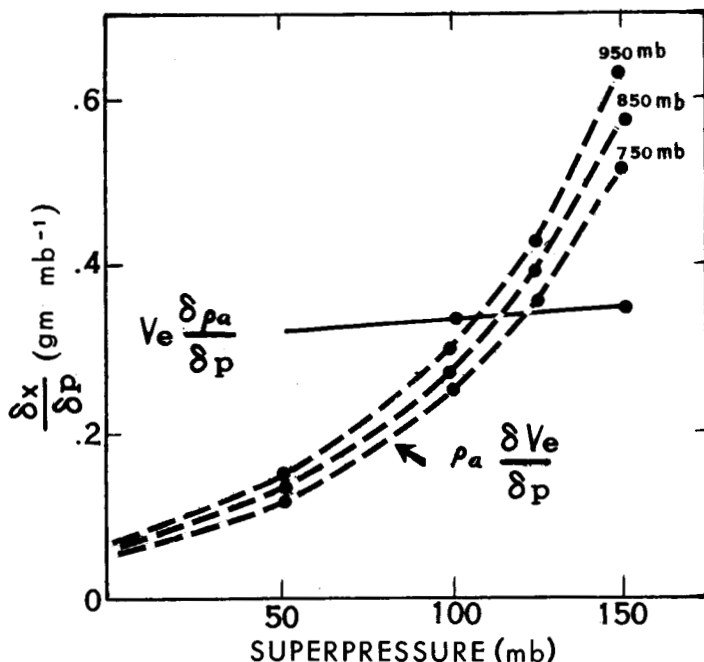


FIGURE 3.—Relative magnitudes of the density-change term (solid line) and the volume-change term for various pressure heights (dashed line) in equation (3) as a function of tetron superpressure (abscissa). The ordinate gives the change in the weight to be attached to the tetron to enable it to fly at a different pressure surface  $\Delta p$  millibars away.

is the superpressure in millibars. Of great interest are the conditions under which the volume change of the 42-in. tetron given by equation (5) is of sufficient magnitude to over-compensate the density-change term in

TABLE 1.—Tetron tracking experiments during 1958-59

Location	Date	Tracked by	Max. range (n. mi.)	Results
Naval Research Laboratory, Chesapeake Bay Annex.	9-58	Mark 25 (3-cm.), ANA/SPQ-2X1 (10-cm.) fire control radar and optical theodolite.	9-----	Good.
Oak Ridge, Tenn. (Oak Ridge National Laboratory).	9-58	Decca 40 (3-cm.) Weather radar (fixed antenna).	?-----	Ambiguous.
Dawsonville, Ga. (Georgia Nuclear Laboratory, Lockheed Aircraft Corp.).	10-58	Decca 40 (3-cm.) Weather radar (fixed antenna).	?-----	Ambiguous.
Idaho Falls, Idaho (National Reactor Testing Station).	11-58	Modified APS-3 (10-cm.) radar.	4-----	Poor.
Oak Ridge, Tenn.-----	3-59	FPS-10 (10-cm.) radar and optical theodolite.	4-6-----	Poor, no radar return.
Manassas, Va. (U.S. Air Force, Air Defense Command).	3-59	FPS-3 (23-cm.) radar.	(Altitude 5,000 ft.) 15. (Altitude 10,000 ft.) 60.	Good.
Hatteras, N.C. (U.S. Weather Bureau station).	9-59 to 10-59	Modified SP-1M (10-cm.) radar.	92-----	Excellent.

equation (3), since if this criterion is satisfied the tetron is no longer a constant level balloon. Figure 3 shows the relative magnitudes of the density-change term (solid line) and the volume-change term for various pressure-heights (dashed lines) in equation (3) as a function of tetron superpressure (abscissa). Since the volume-change term in figure 3 is plotted as the negative of the way it appears in equation (3), it is seen that if the tetron superpressure much exceeds 100 mb., the ordinate,  $\delta X/\delta p$ , becomes negative and the tetron will ascend until it bursts. Consequently, if these tetrons are to be flown at ambient pressures of less than 900 mb., it is necessary to launch them in a partially deflated state. Procedures for determining the weight to be added to the tetron in order to attain flight at a given density surface are given in the appendix for cases when (a) the tetron volume is assumed constant, (b) the flight is to be made at low levels and the tetron volume is assumed to vary according to equation (5), and (c) the tetron is launched in a semi-deflated state for flight at high level and the tetron volume is assumed to vary according to equation (5).

#### 4. SUMMARY OF TETRON FLIGHTS DURING 1958-59

Table 1 gives a summary of tetron flights made through the year 1959. Tetron flights were first made at the Chesapeake Bay Annex of the Naval Research Laboratory, where the attachment of a metalized mesh to the aluminized tetron permitted the balloon to be tracked for 9 mi. Without this mesh the radar returns were sporadic, probably depending upon whether the face or the corner of the tetron was directed toward the radar. Figure 4 shows three altitude-versus-time plots of the most successful tetron flight at the Chesapeake Bay Annex, as obtained from radar information and a combination of radar and visual theodolite information. Of interest in this figure is the evidence that the tetron overshot its floating level and subsequently performed damped vertical oscillations of a period (in this case) of about 20 min.

Later tetron flights at Oak Ridge, Tenn. and Dawsonville, Ga. were not successful. Conditions for tracking were poor with precipitation echoes scattered over the Decca PPI scope. In addition, the ground clutter was bothersome at both stations. Poor results also were obtained from the tetron flights made at Idaho Falls, Idaho, in November 1958, probably owing to the low power of the modified APS-3 radar.

The Air Defense Command (ADC) radar at Oak Ridge was unsuccessful in attempting to track tetrons in March 1959. Again precipitation was present in the vicinity of the radar and it is believed that at least one tetron entered a region of precipitation and was forced to the ground. On the other hand, tetron flights made from the U.S. Air Force ADC site at Manassas, Va. during this same month were moderately successful, and sporadic tracking of a 10,000-ft. flight was carried out for a distance of 60 mi. A longer track might have been obtained except for the fact that the constant level tetrons tend to fly out of the cone of vision of the upward-directed ADC radars. The long strips of aluminized Mylar suspended from the tetron corners on some of the Manassas flights did not noticeably improve the radar reflectivity of the balloon system and proved most unwieldy during launching.

Finally, in September and October 1959, tetron flights were made from the Weather Bureau Station at Cape Hatteras, N.C. Tracking was accomplished by means of the manually operated SP-1M radar of the Weather Bureau. These flights were most successful, and consequently, the remainder of this article is devoted to these flights and the meteorological results obtained therefrom.

## 5. TETRON FLIGHTS FROM CAPE HATTERAS

### A. EQUIPMENT AND PROCEDURES

The SP-1M radar at Cape Hatteras has a power of 750 kw., a wavelength of 10 cm., and a conical beam width of  $3.4^\circ$ . The range accuracy of the SP-1M is stated only as being within 200 yd. If it is assumed that this means that 99 percent of the time the error in range is less than 200 yd., then assuming a Gaussian distribution of range errors, the average range error would be 77 yd. Since the error in the difference between two quantities equals the square root of the sum of the squares of the individual errors, the average error in distance between two radar-determined ranges would be 109 yd. Thus it is estimated that the average SP-1M-determined tetron speed error,  $\Delta V$ , in knots, for a tetron moving radially away from the radar would be given by

$$\Delta V = 3.2/T \quad (6)$$

where  $T$  is the time interval between positions in minutes. One might like to make  $T$  large in order that the error in tetron speed determination be small. On the other hand, the larger  $T$  is chosen, the more the high frequency speed oscillations are damped. The compromise adopted

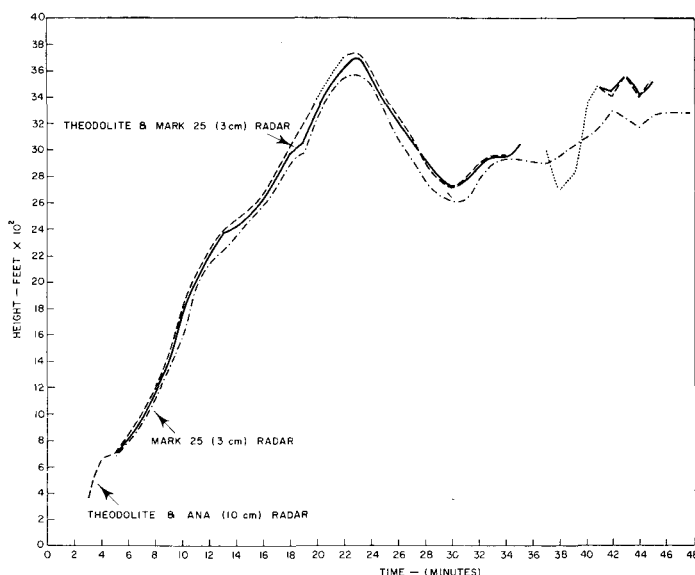


FIGURE 4.—Tetron altitude as a function of time after release for Flight 2 from the Chesapeake Bay Annex.

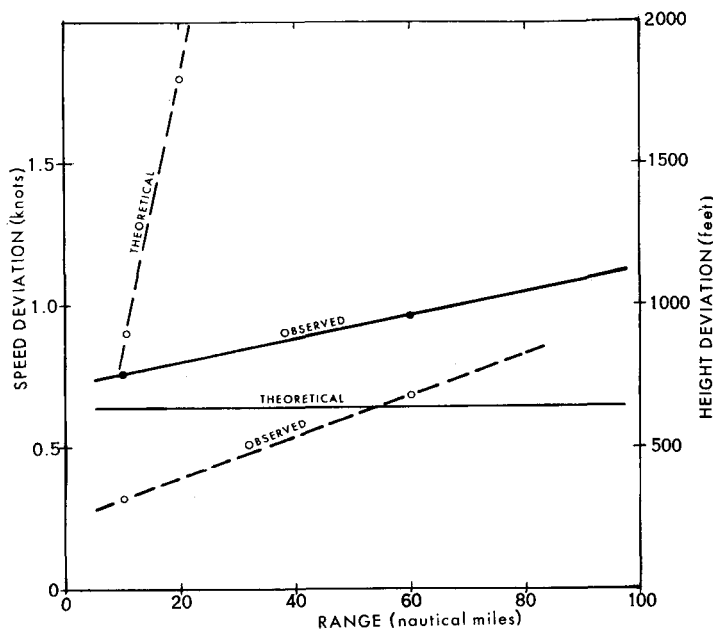


FIGURE 5.—Theoretical average errors in tetron speed (solid line) and height (dashed line) as a function of range for the SP-1M radar at Hatteras. The remaining pair of lines gives the observed average absolute deviation of speed and height from 20-min. average values as a function of range for the tetron flights from Cape Hatteras.

here involved the determination of speed over a 5-min. interval, thus presumably yielding an average speed error of 0.64 kt. and introducing a 50 percent reduction in the magnitude of oscillations of 8-min. period. Figure 5 shows by means of solid lines this theoretical error in wind speed and, for comparison, the regression line of the observed average absolute deviation of tetron speed from the 20-min. average value as a function of radar range. Since there is no reason to expect the actual speed oscillations to increase in magnitude with distance from the



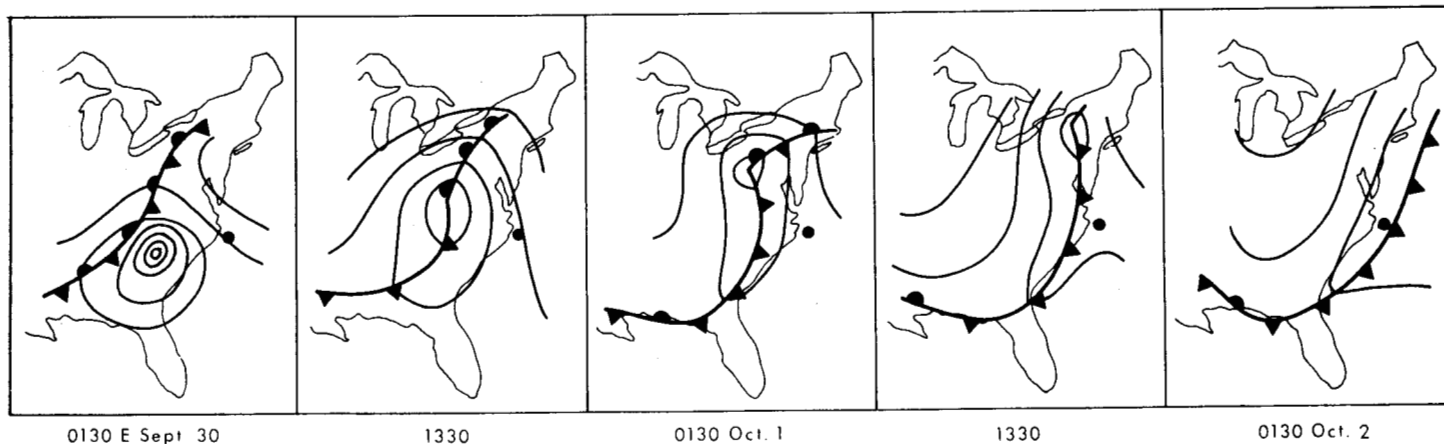


FIGURE 6.—Surface pressure patterns at 12-hr. intervals during the period of tetron flights from Cape Hatteras. The position of the Cape Hatteras Station is indicated by the heavy dot. Isobars drawn at 4-mb. intervals.

radar, the deviation of the observed line from the horizontal probably indicates that actually there is a greater uncertainty in the wind speed determinations as the radar range increases.

Large errors in azimuth and elevation angles may result from the use of the SP-1M radar because of the  $3.4^\circ$  conical beam width (defined as the angular measure of the distance from the power  $P$  at the center of beam to  $0.5 P$ ) and the lack of a precision target indicator (PTI).

If, in the absence of the PTI, it is assumed that it is equally probable the tetron will be positioned anywhere within the beam the maximum angular error would be  $1.7^\circ$  with an average error of  $0.85^\circ$ . The average SP-1M-determined height (or lateral) error  $\Delta H$ , in feet, would be given by

$$\Delta H = 90R \quad (7)$$

where  $R$  is the range in nautical miles. Figure 5 shows by means of dashed lines this theoretical error in height determination and, for comparison, the regression line of the observed average absolute deviation of tetron height from the 20-min. average value as a function of radar range. The observed deviation increases much more slowly than the theoretical, presumably because of the necessity for aligning the target more nearly in the center of the radar beam in order to get a good signal return at great ranges and the subjective tendency not to vary the elevation angle of the radar at these ranges unless forced to do so. The tetron targets were manually tracked, with one man operating elevation and azimuth controls and one on the range scope. The balloon position was read from these dials at 1-minute intervals. In any event, it is obvious that the vertical and cross-stream motions of the tetron obtained from the SP-1M radar should be treated with caution.

One of the purposes of the Cape Hatteras tetron flights was to test the improvement in radar return resulting from use of a metalized nylon mesh obtained from Suchy

TABLE 2.—Tetron flights from Cape Hatteras, N. C.

Flight no.	Time of launching (EST)	Date	Flight duration (hr.)	Max. range (n.mi.)	Average height (100's of ft.)	$\sigma_h$ (100's of ft.)	Average speed (kt.)	$\sigma_s$ (kt.)	Flight equipment
1	1108	9-30-59	1.7	52	11	(*)	30.6	(*)	42-in. tetron.
2	1422	9-30-59	4.0	92	42	22	23.0	3.3	42-in. tetron with 10-cm. radar mesh.
3	2037	9-30-59	1.9	50	16	8	26.3	2.7	60-in. tetron with 3-cm. radar mesh.
4	0825	10-1-59	4.4	60	40	16	13.6	1.7	Train of 3 42-in. tetroons.
5	1413	10-1-59	5.0	51	143	22	10.2	2.3	60-in. tetron.

\*Not computed—tracking sporadic.  
 $\sigma_h$  = Standard deviation of height.  
 $\sigma_s$  = Standard deviation of speed.

Division, Inc. of New York. This mesh was formed into a cylinder and hung like a skirt from the mid-section of the tetron. The mesh weighed only 50 gm. and provided an additional radar-reflecting area of 30 ft.<sup>2</sup>. Furthermore, the mesh skirt, hanging in the form of a cylinder, greatly lessened the problem of signal fading associated with the shape of the tetron. The success of the mesh can be judged from the fact that the two tetroons on which it was installed were tracked over the radar horizon.

#### B. TETRON TRAJECTORIES

Figure 6 shows the surface pressure pattern at 12-hr. intervals during the period of tetron launchings from Cape Hatteras. The flights were made (inadvertently) while hurricane Gracie was in the process of being transformed into an extratropical storm over the Middle Atlantic States. Consequently, at least 5 min. tracking time was lost each hour due to use of the radar for synoptic observations. Table 2 gives information on the five successful tetron flights made during this period. From comparison of the tetron release time, as presented in the table, and the synoptic maps one can visualize the synoptic situation during the tetron flight. It is seen

from table 2 that in addition to tetroons with metalized mesh attached, tracking was also performed on a plain 42-in. tetroon, a plain 60-in. tetroon, and on three 42-in. tetroons tied together.

Figure 7 shows the trajectories of the five flights. The tetroon positions are plotted at 10-min. intervals while the tetroon velocity, determined from the distance and direction between successive positions, is indicated in conventional meteorological form (in knots). The numbers beneath the positions indicate the tetroon height to the nearest 1,000 ft. All flights except Flight 5 were ballasted to float at 900 mb. (about 3,000 ft.), while Flight 5 was ballasted to float at 700 mb. (about 10,000 ft.). The error in altitude determination of Flight 5 resulted from the assumption that equation (5) applied to 60-in. tetroons as well as to 42-in. tetroons for which it was derived. It is noted that near the end of the trajectories on Flights 2 and 4 there was a sudden jump in the tetroon height as indicated by the radar. Since it is most unlikely that the tetroons appreciably changed altitude, it is probable that these height jumps depict the complex refraction and reflection phenomena of the radar beam near the radar horizon. Therefore, while the horizontal trajectory of the tetroon may be determined near to and even beyond the radar horizon, it is probable that the vertical positions of the tetroon indicated by the radar near the radar horizon are of little significance. If, however, the variations in signal are random, spectral analysis can separate significant vertical motions. Even if the signal variations are ordered there is the possibility, by comparison with data closer to the radar, of identifying the spurious variations due to anomalous propagation.

### C. VELOCITY FLUCTUATIONS

Figure 8 shows the speed as a function of time as obtained from the four accurately positioned tetroon flights. This speed was determined only from the change in radar range apropos the discussion in 5a concerning the inaccuracy of azimuth angles determined by the SP-1M radar. The dashed speed traces indicate places where the tetroon speed had to be interpolated, either due to use of the radar for synoptic purposes or to loss of contact with the tetroon through a fading radar signal. The vertical arrows near the beginning of each trace indicate the approximate time the tetroons reached flight altitude. Most apparent in figure 8 is the long term increase of wind speed with time on Flights 2 and 3. A glance at the first 1330 EST map in figure 6 shows that this speed increase is in agreement with the increase of surface pressure gradient as one moves north of Cape Hatteras. On Flight 5 there was a most surprising periodicity in wind speed near the beginning of the flight. Several times the wind speed changed by 6 kt. in about 13 min. of time (2.8 n. mi. travel distance) with some evidence that the speed increase was more abrupt than the speed decrease. More will be said about this periodicity later.

The significance of the speed changes shown in figure 8 can be estimated from the discussion of the range accuracy of the SP-1M radar and the deviation of the wind speed from the 20-min. averages presented in section 5a.

For each of the four well-positioned tetroon flights, the contribution of oscillations of various frequency to the variance of the series was determined for the wind speed,  $V$ , the along-stream,  $V_s$ , and cross-stream,  $V_n$ , velocity components (where the "stream" is defined by the total wind vector for a run) and the vertical motion,  $W$ , in the manner indicated by Tukey [14]. Spectra were determined separately for both the earlier portion of the flight, when little interpolation of the velocity values was required, and for the entire flight. These spectra were then averaged so that the "good" portion of the flight was weighted twice as heavily as the "poor" portion of the flight. While considering the spectral and autocorrelation curves so obtained it should be kept in mind that the "predominant" periodicities which result are to some extent related to the interval and length of sampling, and furthermore that the fraction of the energy of the velocity represented by  $V_s$  and  $V_n$  at a given frequency is in part a function of the orientation of the coordinate axes. The degree to which these factors bias the results presented herein will be more readily apparent upon analysis of the precise trajectory data obtained by means of tetroon flights from the Wallops Island station of the National Aeronautics and Space Administration.

The variation with frequency of the speed variance for the individual flights, as well as the mean variance for all the flights, is indicated in the top diagram of figure 9 for periods of oscillation varying from 3 hr. to 6 min. The greatest speed variance is associated with the low frequency oscillations, which are probably "synoptic" or inertial in character. Flights 5 and 3 yield a pronounced tendency for speed fluctuations of period near 26 min. These two flights have sufficient influence on the mean to make the next most pronounced mean periodicity one of 26-min. period. Less pronounced peaks in the mean spectra occur at periods of 12-13 min. and 8 min. Since the speed has been averaged over a 5-min. interval, the spectral peak at 12-13 min. might be due to errors in range determination.

The variation with frequency of the variance of the vertical tetroon motion is given in the lower diagram of figure 9. With the exception of minor peaks of dubious significance, the mean variance of tetroon vertical motion is at a maximum near a period of 13 min. The three possibilities with regard to this peak are: (a) the 13-min. period in tetroon oscillation represents a similar period in vertical air motions, (b) the 13-min. period reflects only vertical tetroon oscillations and does not reflect the vertical air motions at all, and (c) the 13-min. period does not even represent the predominant period of tetroon oscillations but is indicative only of errors in the elevation angles obtained from the SP-1M radar (see section 5a.).

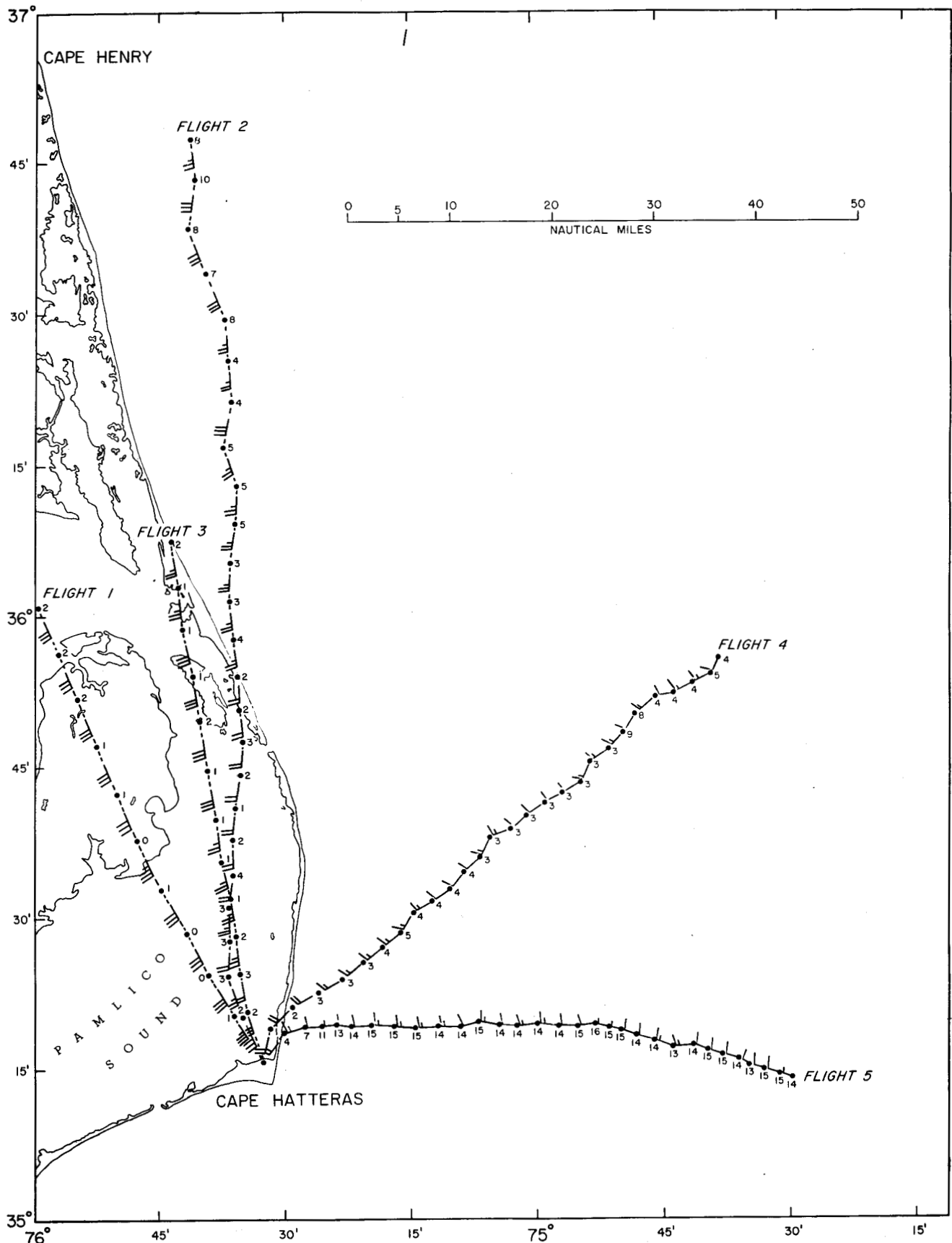


FIGURE 7.—Trajectories of the five tetron flights from Cape Hatteras. The tetron positions are plotted at 10-min. intervals, while the tetron velocity (in knots) is indicated in the conventional meteorological form. The tetron height in thousands of feet is indicated beneath each position.



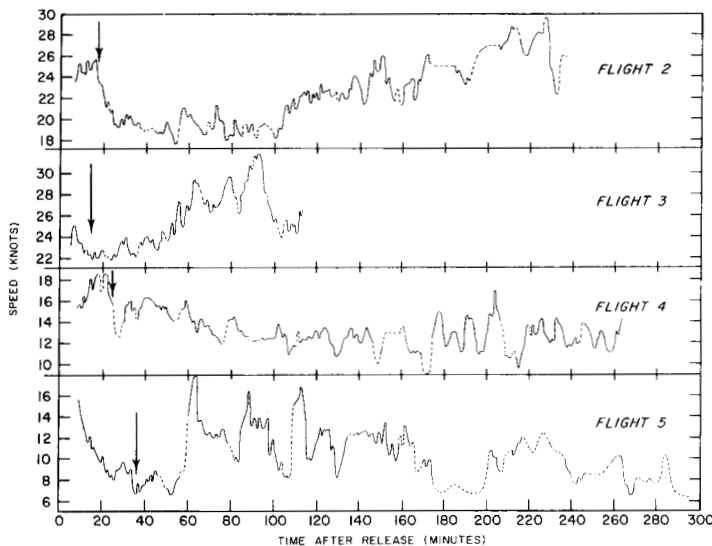


FIGURE 8.—Wind speed as a function of time after release as obtained from the four accurately positioned tetron flights from Cape Hatteras. The dashed traces indicate interpolated speed values. The arrows indicate the approximate time the tetrons reached flight altitude.

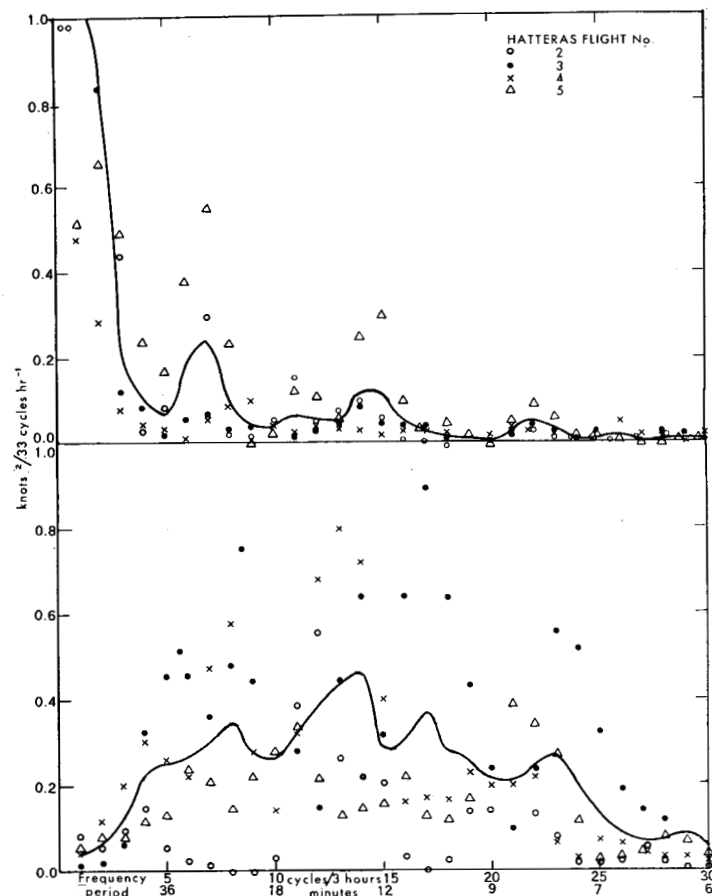


FIGURE 9.—Speed variance (top) and vertical velocity variance (bottom) as functions of frequency of oscillation for individual tetron flights and the mean of all flights (solid line) from Cape Hatteras.

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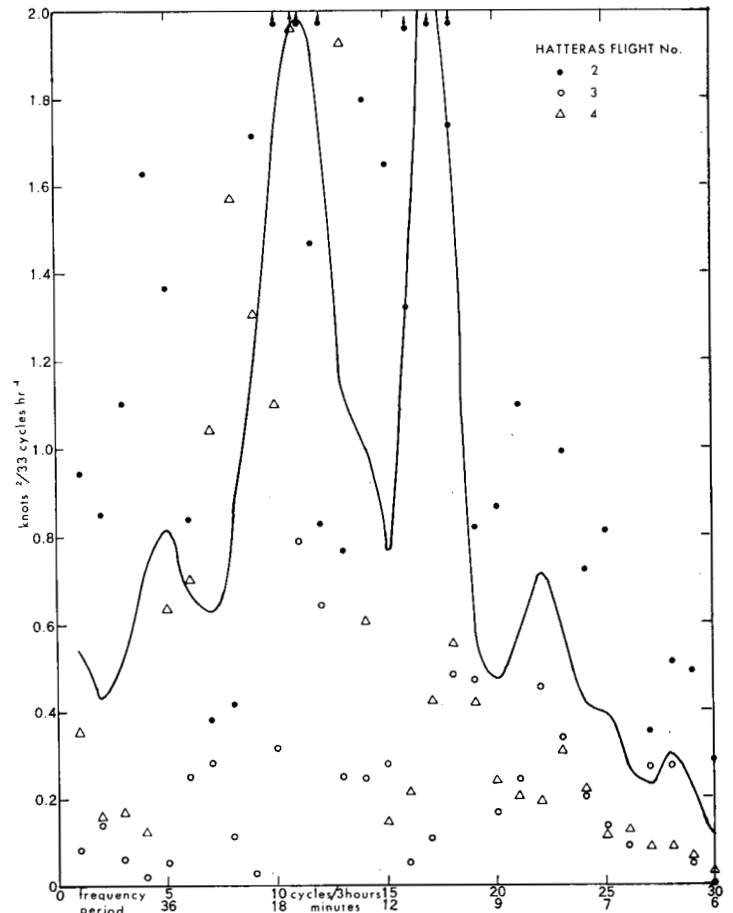


FIGURE 10.—Cross-stream velocity variance as a function of frequency of oscillation for individual tetron flights and the mean of all flights (solid line) from Cape Hatteras.

Discussion of these possibilities is reserved for the next subsection. It is interesting to note, however, that excluding the variance associated with oscillations of more than a 1-hr. period the flights with relatively large values of the speed variance have relatively small values of the vertical motion variance. Thus Flights 3 and 5 each have a relatively large speed variance (1.9, 4.1 kt.<sup>2</sup>) and relatively small vertical velocity variance (2.7, 2.8 kt.<sup>2</sup>) while Flights 2 and 4 each have a relatively small speed variance (1.0, 0.7 kt.<sup>2</sup>) and a relatively large vertical velocity variance (10.0, 6.8 kt.<sup>2</sup>). In considering these values it should be remembered that Flight 3 was an evening flight, while Flight 5 was a flight at 14,000 ft. In any event, this negative correlation between the magnitudes of speed and vertical velocity variances suggests that most of the speed variance is not due to vertical oscillations of the tetrons in regions of vertical wind shear.

Figure 10 shows the variance of the cross-stream velocity component as a function of frequency. Note that at all frequencies the variance of the cross-stream velocity component is very large compared to that of the speed.

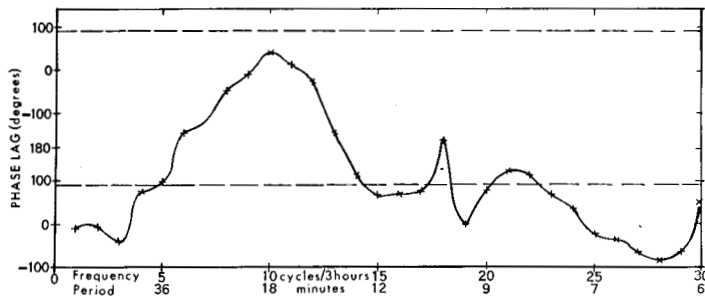


FIGURE 11.—Mean phase lag between tetron vertical motion and speed as a function of frequency of oscillation for the Cape Hatteras flights. The dashed lines drawn at a phase lag of  $90^\circ$  indicate where the maximum upward tetron motion followed the maximum tetron speed by a phase lag of  $90^\circ$  ( $\frac{1}{4}$  wavelength).

This is due to uncertainties in azimuth determination by the SP-1M radar. Of the two peaks in the cross-stream velocity variance, the one at a period near 10 min. is probably due to azimuth errors while the one at a period near 17 min. may reflect a real periodicity in the atmosphere during this time.

The variance of the along-stream velocity component was also determined as a function of frequency. However, since it so closely mirrors the results presented in figure 9 for the speed, it is not reproduced here. It should be mentioned that Mantis [13], from analysis of accurately positioned superpressured balloon flights at 30,000 ft., found a minimum of velocity variance at periods of 12–24 min., certainly not in agreement with the results found here.

#### D. CONSIDERATIONS OF TETROON MOTIONS AS INDICATORS OF AIR MOTIONS

In estimating the degree to which oscillations in the tetron velocity represent oscillations in air velocity, it is desirable to have knowledge of the phase lag between the oscillations of various velocity components. This phase lag as a function of frequency of oscillation may be obtained from evaluations of the co-variance and quadrature variance in the manner indicated by Van der Hoven and Panofsky [16].

Let us first consider the significance of the speed periodicities indicated by the upper diagram of figure 9. It is apparent that if the tetron does not follow the vertical air motion but instead, perhaps due to overshooting its flight level initially (fig. 4), undergoes vertical oscillations independent of vertical air motions in a region of vertical wind shear, then a fictitious periodicity in wind speed will be introduced. We should therefore be suspicious of any wind speed periodicity which is associated with a periodicity in vertical tetron motion. From figure 9 this association is most obvious for oscillations of 12–13-min. period. For further evidence we turn to the phase lag between wind speed and vertical tetron motion (fig. 11). Since, for all four of the tetron flights under consideration, the wind speed was decreasing

with elevation, a tetron performing vertical oscillations independent of vertical air motions would show the maximum speed preceding the maximum upward motion by a phase lag of about  $90^\circ$ . In figure 11 the dashed horizontal line indicates this phase difference and it is seen that oscillations of 40-min. period and oscillations of period 13–8 min. possess this  $90^\circ$  phase difference. Therefore the 12–13-min. periodicities in the wind speed may well be due to vertical oscillations of the tetron in a region of wind shear. On the other hand, the 26-min. periodicity in the wind speed is certainly not due to vertical oscillations of the tetron, since the maximum wind speed follows rather than precedes the maximum upward motion with a phase difference of about  $90^\circ$  for oscillations of 26-min. period.

The above discussion, while suggesting that variations in tetron speed of 12–13-min. period are due to vertical oscillations of the tetroons with this period, does not prove that the vertical oscillations were tetron oscillations only and not those of the air, since a tetron enclosed in a blob of air would also experience, although to a lesser extent, the component phase lags mentioned above. Therefore, we next determine whether the predominant period of vertical tetron oscillation is reasonably close to the period which would be expected of an air parcel. By relating the vertical acceleration of an air parcel to the buoyancy force acting upon it, it can be shown that the period of vertical oscillation ( $\tau$ ) of an air parcel in the atmosphere is approximately given by

$$\tau = 2\pi \sqrt{\frac{T_0}{g(\gamma_p - \gamma)}} \quad (8)$$

where  $T_0$  is the absolute temperature in the vicinity of the air parcel,  $g$  is the acceleration of gravity,  $\gamma_p$  is the process lapse rate (usually assumed dry adiabatic) and  $\gamma$  is the lapse rate [9]. This equation is not exact, since the effects of the surrounding air are not considered. These effects tend to make the period of oscillation greater, the more so the greater is the horizontal dimension of the cell in relation to its vertical extent. For example, in cells with the dimensions of Bénard cells (cell height three times cell diameter), the period of vertical oscillation would be 1.46 times the period given by equation (8) [5]. Since it is impractical to estimate the cell dimensions from the tetron flights it must suffice to say, that if the tetron follows the 3-dimensional path of an air parcel, the period of vertical oscillation of the tetron should be between  $\tau$  and  $1.5\tau$ . Table 3 gives, for the 4 flights, the period of maximum variance of tetron oscillation in the vertical, as obtained from the lower diagram of figure 9, and the period computed from equation (8) by utilizing radiosonde ascents at Cape Hatteras to yield information on the mean lapse rates and temperatures at the average floating level of the tetroons. In all cases the predominant periodicity in tetron oscillation in the vertical exceeded that given by equation (8), with the multiplication factor being 1.1 on

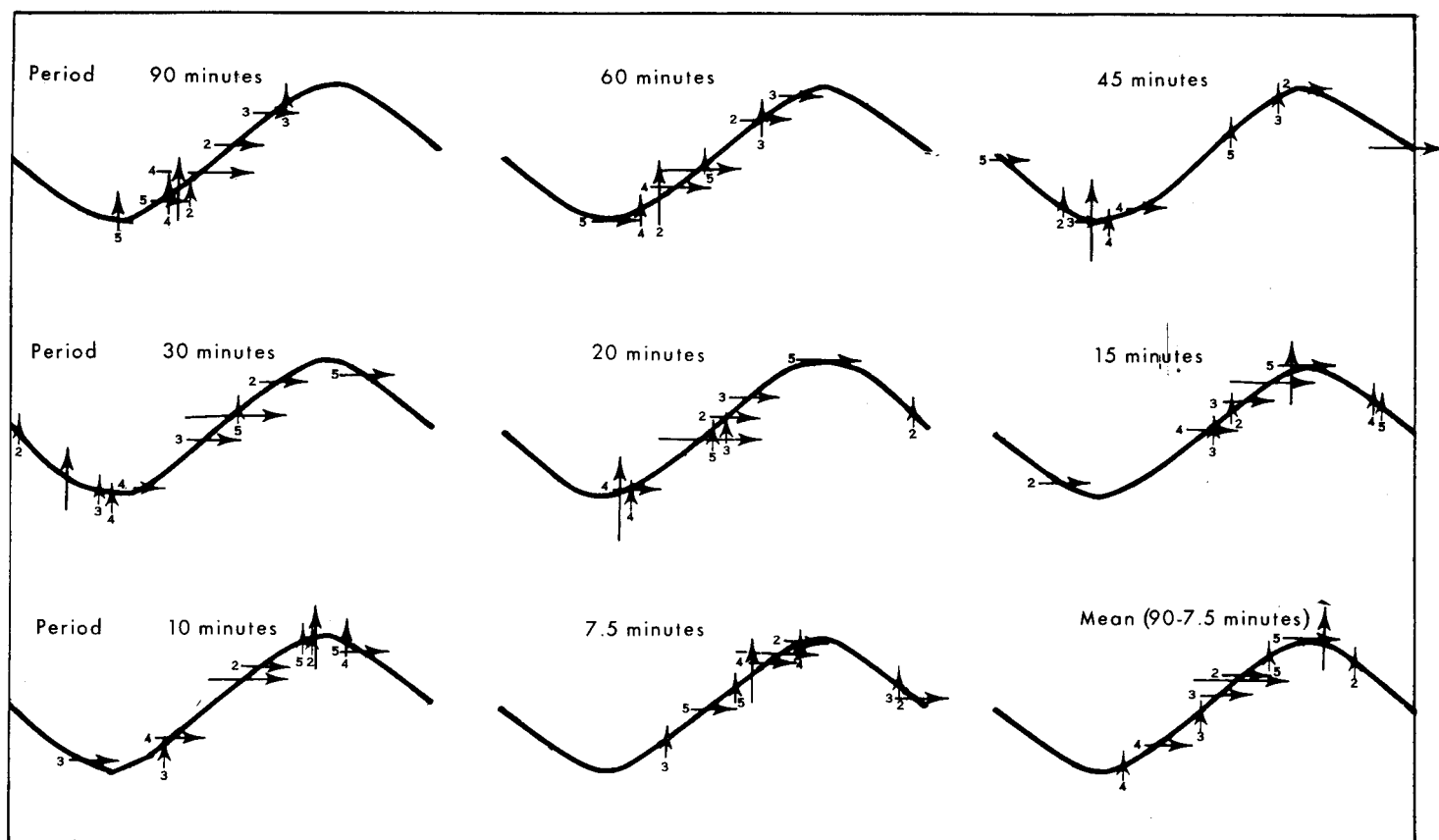


FIGURE 12.—Positions of maximum tetron speed (horizontal arrows) and maximum tetron upward motion (vertical arrows) relative to tetron transversal velocity components (schematic wave-shaped trajectories) for oscillations of various periods (in minutes) for the individual numbered flights and the mean of all flights (long arrows) from Cape Hatteras.

Flights 2 and 5, 1.3 on Flight 3, and 1.8 on Flight 4. On the other hand, the natural period of oscillation of an absolutely constant volume tetron (neglecting aerodynamic drag which, for the tetrahedrons, must be a complicated (and unknown) non-linear function of the restoring buoyancy force) in an atmosphere with the given stability would be about one-fourth the observed 13-min. period of vertical oscillation, while the period of vertical oscillation of a completely elastic helium-filled balloon (helium lapse rate equals  $1.3^{\circ}$  C. per 100 m.) would be about one-half the observed predominant period. Thus the predominant period of 13 min. is not due to natural

vertical oscillations of the tetron and the hypothesis that the tetron at least partially follows the vertical air motion is certainly not contradicted except perhaps in the case of Flight 4. However, since this latter flight consisted of 3 tetroons tied together (which probably had different natural floating levels), it would not be surprising to find vertical air motions poorly represented by the vertical motions of this tetron train.

#### E. PHASE LAG BETWEEN CROSS-STREAM FLOW, AND MAXIMUM SPEED AND UPWARD MOTION FOR OSCILLATIONS OF DIFFERENT PERIODS

Figure 12 gives the position of maximum wind speed (horizontal arrows) and maximum upward motion (vertical arrows) along schematic wave-shaped trajectories for each flight, and the mean of all flights, for various periods of oscillation. In the mean for all flights it is seen that for most oscillation periods the speed was at a maximum ahead of the trough in the trajectory, the only exception being oscillations of a 45-min. period. Thus, if it is assumed that the large-scale air flow is nearly geostrophic and that small-scale geostrophic oscillations do not exist, then the tetron moved faster when it was moving down the pressure gradient than when it was mov-

TABLE 3.—Comparison of predominant period of tetron vertical motion and theoretical period of vertical air motion for Cape Hatteras flights

Flight	Predominant tetron period (min.)	Period computed from equation (8) (min.)
2	10.6	9.4
3	15.0	11.2
4	13.8	7.7
5	8.6	8.0

TABLE 4.—Autocorrelation coefficients of tetron speed ( $V$ ), vertical motion ( $W$ ), cross-stream ( $V_n$ ) and along-stream ( $V_s$ ), velocity components for Cape Hatteras flights

Minutes	$R(V)$	$R(W)$	$R(V_n)$	$R(V_s)$	$\frac{1-R(V_s)}{1-R(V_n)}$
1	0.90	0.61	0.69	0.87	0.42
2	.81	.32	.40	.76	.40
3	.72	.12	.15	.67	.39
4	.62	-.10	-.10	.57	.39
5	.55	-.20	-.22	.49	.42
6	.52	-.23	-.27	.45	.43
7	.51	-.14	-.23	.45	.45
8	.50	-.10	-.18	.45	.47
9	.48	-.08	-.13	.46	.48
10	.47	-.07	-.06	.47	.50
12	.44	-.02	.01	.48	.53
14	.42	.02	.05	.45	.58
16	.38	.04	.09	.39	.67
18	.33	-.03	.09	.31	.76
20	.32	-.11	.04	.27	.76
24	.36	-.01	-.10	.31	.63
28	.23	.04	.01	.19	.82

ing up the pressure gradient. Thus, there is some evidence that kinetic energy (and momentum) is transported down the pressure gradient by these small-scale oscillations. Moreover, in the mean for all flights, the maximum upward motion of the tetron took place near the trough line for oscillations of a period exceeding 18 min. and near the trajectory crest for oscillations of a smaller period. Thus, looking downstream the long-period tetron oscillations were counterclockwise in a plane normal to the trajectory, while the shorter-period oscillations were clockwise. Insofar as the isotherms are parallel to the contours at this elevation, with cold air to the left of the flow looking downstream, this means that the long-period tetron oscillations are direct (warm air rising, cold air sinking) while the short-period oscillations are indirect (cold air rising, warm air sinking). However, since, according to figure 11, the short-period tetron oscillations in the vertical may not be representative of air oscillations in the vertical, the evidence for indirect circulations should be treated with caution. Further, until data on the temperature field are available on a comparable scale with the air motions here discussed, comparisons derived from synoptic-scale experience must be quite tentative.

#### F. SOME COMPARISONS WITH OTHER LAGRANGIAN DATA

Mean autocorrelation coefficients of the speed,  $V$ , vertical velocity,  $W$ , and cross-stream,  $V_n$ , and along-stream,  $V_s$ , components of tetron velocity for the four Cape Hatteras flights are presented in table 4 for time lags from 1 to 28 min. Also presented in this table is the ratio of one minus the along-stream and one minus the cross-stream autocorrelation coefficients. It can be shown [10] that for Eulerian space correlation functions in the inertial range this ratio would be expected to be 0.6 for two-dimensional isotropic turbulence. It is of interest that in table 4 this ratio, while fluctuating between 0.4 and 0.8, does have a mean near 0.6, suggesting a certain similarity between Eulerian space and Lagrangian autocorrelation coefficients.

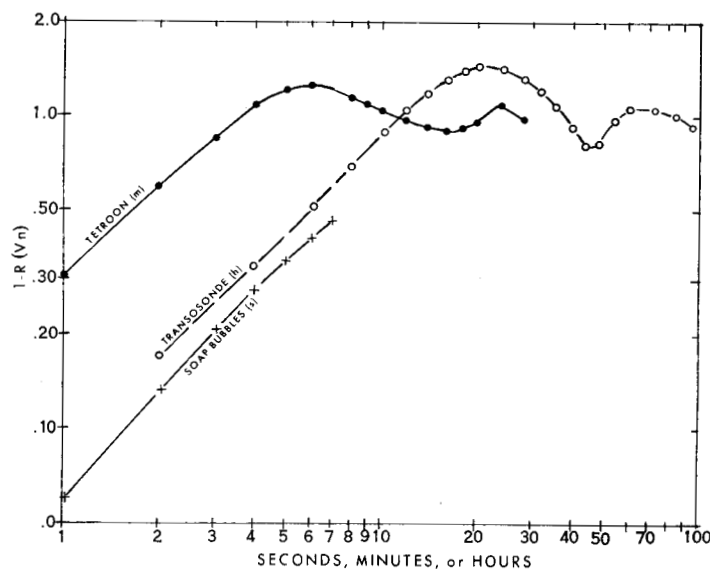


FIGURE 13.—Comparison of autocorrelation coefficients of the cross-stream velocity component obtained from Cape Hatteras tetron flights, 300-mb. transosonde flights, and the soap bubble data of Edinger. (Note that the data are plotted on log-log paper, that the ordinate is in terms of one minus the autocorrelation coefficient, and that the abscissa is minutes, hours, and seconds, respectively, for the tetron, transosonde, and soap bubble data.)

In figure 13 the value of one minus the cross-stream autocorrelation coefficient for the tetron flights is plotted as a function of time on log-log paper. For purposes of comparison with other Lagrangian time scales, figure 13 also shows similar curves for the cross-stream velocity components as obtained from transosonde data [3] and the soap bubble data of Edinger [4]. Note that the abscissa in this figure represents seconds for the soap bubble data, minutes for the tetron data, and hours for the transosonde data. It is seen that initially all three curves are approximately straight lines with slopes near  $45^\circ$  on the log-log plot, showing that for all three scales of motion  $1-R(V_n) \propto t$  as would be anticipated from Lagrangian turbulence theory [11]. However, since pronounced periodicities are present in the cross-stream wind component along both the tetron (17-min. period) and transosonde (46-hr. period) trajectories, the above proportionality between time and  $1-R(V_n)$  soon breaks down. The soap bubbles could not be tracked for a length of time sufficient to delineate the predominant periodicity in that small-scale flow. Replacing the sign of proportionality by a sign of equality, we find that initially the autocorrelation coefficient for the tetron data is given by

$$R(V_n) = 1 - 4.9t/p_0 \quad (9)$$

while for the transosonde data it is given by

$$R(V_n) = 1 - 3.7t/p_0 \quad (10)$$

where  $p_0$  represents the predominant periodicities of oscillation (17 min., 46 hr.) cited above. These values for the constants differ from the theoretical Eulerian-time constants deduced by Ogura (1.67) and Gifford (1.18), and reported by Gifford [6], as would be expected. An interesting point is the ratio of these constants, 4.15 for the tetroons and 3.14 for the transosonde flights. Hay and Pasquill [8] have related Eulerian and Lagrangian turbulence statistics (on the assumption that the correlations decay with time in the same manner) by the factor  $\beta$ . They tabulated  $\beta$  values for a large range of scales together with the associated turbulence intensity. The scale of our experiments is intermediate between the diffusion experiments and the air trajectories they examined, and computed turbulence intensities for the tetroons (table 5) are also of the right order to fit this intermediate scale. Thus, these data do not contradict the use of a scale factor to relate Lagrangian and Eulerian data and indeed support the selection of a  $\beta$  value near 4.

It is also interesting to reverse the usual procedure and to work from these Lagrangian statistics to an estimate of Eulerian statistics. Following Gifford's [7] equation (6), one can estimate the position of the Eulerian spectral maximum. Such estimates from the tetroon flights indicate Eulerian maxima in the cross-wind component,  $V_n$ , between 12 and 43 cycles per hour, a frequency not in disagreement with the high frequency peak computed by Van der Hoven [15] for the Brookhaven tower data. The comparison suffers from a great difference in the height of the observations but it is, at least, not contradictory. The peak in the vertical spectra can be shifted in the same manner, giving Eulerian peaks at frequencies from 37 to 68 cycles per hour. Comparison of these values with those obtained by Jones [12] from wind inclination data at 2,000 ft. again shows reasonable agreement. (However, Jones' data do not extend far enough into the low frequency range to completely position his spectral peak.)

Taking into account the difficulties in positioning the tetroons previously discussed, one can examine the total variances in relation to any assumption of isotropy. From table 5 it is seen that in no case were the variances in the three components equally distributed and the lateral,  $V_n$ , to vertical,  $W$ , ratio ranged from about 2 to 4. Flight 3 is particularly interesting in this regard. It was the only nocturnal flight and indicates both reduced turbulence and a smaller  $V_n/W$  ratio.

## 6. CONCLUSION

The foregoing shows that data of interest and importance can be obtained from low-level tetroon flights even when the radar tracking is not too precise. During January 1960, eight tetroon flights were tracked by the excellent FPS-16 radar at the Wallops Island station of the National Aeronautics and Space Administration. The FPS-16 radar positions the tetroons with a root mean square error of only 5 yd., and for these flights radar

TABLE 5.—Variance distribution and turbulence intensities

Flight No.	Total variance (kt. <sup>2</sup> )			Average wind speed (kt.) $\bar{V}$	Turbulence intensity			
	$\sigma_{V_n}^2$	$\sigma_{V_z}^2$	$\sigma_W^2$		$\sigma_{V_n}/\bar{V}$	$\bar{V}/\sigma_{V_n}$	$\sigma_W/\bar{V}$	$\bar{V}/\sigma_W$
2-----	40.8	4.0	10.6	23.0	0.28	3.6	0.14	7.1
3-----	6.9	7.8	3.1	26.3	.10	10.1	.07	14.9
5-----	18.7	5.6	4.9	10.2	.42	2.4	.22	4.6
Average-----	25.2	5.4	6.9	*17.7	.28	3.5	.16	6.8

\*Weighted Average.

range, azimuth, and elevation data were available at 10- or 30-sec. intervals. Consequently, information can be obtained from these flights on high frequency Lagrangian oscillations that could not possibly be obtained from the SP-1M radar. Thus, with the inclusion of transosonde data, information becomes available (admittedly at different elevations) concerning Lagrangian fluctuations with periods from a few tens of seconds to a week, with only a slight gap at periods of 3-8 hr. It is hoped that by passing tetroons from one radar station to another, reliable information can also be obtained on the Lagrangian fluctuations in this range. Such flights are next on the agenda. In addition, work is progressing on the development of a very light-weight transponder which can be attached to the tetroons in order to increase the tracking range. The future of the tetroon in air pollution studies, and other studies where knowledge of the Lagrangian wind fluctuations is essential, appears bright.

## ACKNOWLEDGMENTS

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## APPENDIX

## A. Weigh-off procedure assuming tetraoon of constant volume

1. Determine surface tetraoon volume ( $V_s$ ) from equation (4).
2. Determine density surface ( $\rho_a$ ) at which flight is to be made.
3. Determine helium density ( $\rho_{hs}$ ) at earth's surface from equation of state for helium.
4. Weight to be added to tetraoon equals  $V_s(\rho_a - \rho_{hs}) - W_b$  where  $W_b$  is weight of tetraoon.

## B. Weigh-off procedure for low-level tetraoon flights assuming tetraoon volume varies according to equation (5)

1. Determine surface tetraoon volume ( $V_s$ ) from equation (4).
2. Determine density surface ( $\rho_a$ ) at which flight is to be made.
3. Determine tetraoon superpressure ( $\Delta p$ ) at density surface of flight assuming tetraoon released in fully inflated state.
4. Determine tetraoon volume at flight level ( $V_e$ ) from  $V_e = V_s + .0038(e^{0.015\Delta p} - 1)$
5. Weight to be added to tetraoon equals  $V_e\rho_a - V_s\rho_{hs} - W_b$  where  $\rho_{hs}$  is surface helium density and  $W_b$  is tetraoon weight.

## C. Weigh-off procedure for high-level tetraoon flights (tetraoon partially deflated at the earth's surface) assuming tetraoon volume varies according to equation (5)

1. Determine surface tetraoon volume ( $V_s$ ) from equation (4).
2. Determine density surface ( $\rho_a$ ) at which flight is to be made.

3. Determine what volume of helium ( $V_h$ ) would fully inflate tetraoon at flight level plus  $\Delta p$ , where  $\Delta p$  is less than 100 mb.:

$$V_h = \frac{V_s \rho_a + \Delta p}{\rho_{as}}$$

where  $\rho_{as}$  is surface air density and  $\rho_{a+\Delta p}$  is air density when tetraoon fully inflated.

4. Deflate tetraoon until free lift of tetraoon equals

$$V_h(\rho_{as} - \rho_{hs}) - W_b$$

where  $\rho_{hs}$  is surface helium density and  $W_b$  is tetraoon weight.

5. Determine tetraoon volume at flight level ( $V_e$ ) from

$$V_e = V_s + 0.0038(e^{0.015\Delta p} - 1)$$

6. Weight to be added to tetraoon equals  $V_e\rho_a - V_h\rho_{hs} - W_b$

## Notes:

1. The selection of a desired flight altitude cannot be entirely arbitrary for the "deflated" tetraoon (Method C) since enough free lift (contained helium) must be retained to escape surface turbulence and clear obstacles in the launching area.
2. There is evidence that marked temperature inversions can inhibit, or even prevent, a "deflated" tetraoon from achieving the chosen flight level by radically altering the air vs. helium density relationships compared to surface inflation conditions. This phenomenon is under study.